“Power-Law Gels, Scott-Blair and the Fractional Calculus of Soft Networks and Multiscale Soft Materials”

Summary

Many soft materials including foods, consumer products, biopolymer gels & associative polymer networks are characterized by multi-scale microstructures and exhibit power-law responses in canonical rheological experiments such as Small Amplitude Oscillatory Shear (SAOS) and creep. Even in the linear limit of small deformations it is difficult to describe the material response of such systems quantitatively within the classical framework of springs and dashpots - which give rise universally to Maxwell-Debye exponential responses. Instead empirical measures of quantities such as ‘firmness’, ‘tackiness’ etc. are often used to describe and compare material responses. G.W. Scott Blair, who was present with Bingham at the very beginnings of the Society of Rheology, argued that such measures are best thought of as ‘quasi-properties’ that capture a snapshot of the underlying dynamical processes in these complex materials. We show that the language of fractional calculus and the concept of a ‘spring-pot’ element provide a useful ontological framework that is especially well suited for modeling and quantifying the rheological response of power-law materials. We illustrate the general utility of this approach by describing fractional differential forms of the Maxwell and Kelvin-Voigt models and using these models we quantify small-amplitude oscillatory shear responses and creep response in range of soft materials including gluten gels, skin and soft tissue, filled polymer melts, hydrogen-bonded biopolymer networks and the complex interfacial rheological properties of acacia gum and serum albumins. The fractional exponents that characterize the dynamic material response can also be connected directly with the scaling exponents from microstructural models such as the Rouse model and the Soft Glassy Rheology (SGR) model. Having determined the quasi-properties that quantify the linear viscoelastic material response of a power-law gel in a concise form, we show that a fractional K-BKZ framework combining a Mittag-Leffler relaxation kernel with a strain-damping function can be used to quantitatively describe the nonlinear viscometric properties of such materials. Depending on the range of values of the quasi-properties the resulting models can have some surprising features, including agreement with well-known heuristics such as the Cox-Merz rule and the complete absence of a zero-shear-rate plateau in the viscosity and the first normal stress difference.